13. APPROXIMATION OF FUNCTIONS

Problem statement: Approximation of given values $\{x(i), y(i)\}_{i=1}^{N}$ by a function $f(\mathbf{a}, x)$ of parameters $\mathbf{a} = \{a(j)\}_{j=1}^{M}$

Solution:

1. The choice of the type of approximation function
   - Linear combination of base functions: $f(\mathbf{a}, x) = \sum_{j=1}^{M} a(j)\, g_j(x)$
   - Nonlinear function
2. Minimization of the mean square error:

$$S(\mathbf{a}) = \sum_{i=1}^{N} (f(\mathbf{a}, x(i)) - y(i))^2 \qquad (1)$$

3. Evaluation of constants $\mathbf{a}$ of the approximation function $f(\mathbf{a}, x)$

13.1 The Mean Square Error Method

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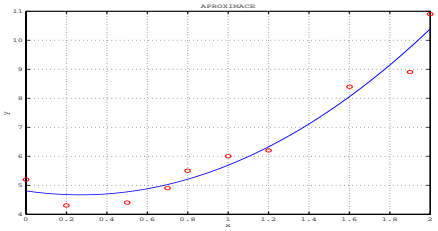
%%% Example 13.1: Derive normal equations for approximation of values (x(i),y(i))
%%% for i=1,..,N by a function f(x)=a(1)*x^M+a(2)*x^(M-1)+...+a(M)*x+a(M+1), M=2
% Definition of given values
    x=[0 0.2 0.5 0.7 0.8 1 1.2 1.6 1.9 2]';
    y=[5.2 4.3 4.4 4.9 5.5 6 6.2 8.4 8.9 10.9]';
% Definition of the matrix of normal equations and its solution
    A=[sum(x.^4) sum(x.^3) sum(x.^2);...
       sum(x.^3) sum(x.^2) sum(x);...
       sum(x.^2) sum(x)    length(x)];
    b=[sum(x.^2.*y) sum(x.*y) sum(y)]'; a=inv(A)*b
% Plot of given and approximation values
    xx=[0:0.05:2]'; f=a(1)*xx.^2+a(2)*xx+a(3); plot(x,y,'or',xx,f);
    grid on; xlabel('x'); ylabel('y'); title('APPROXIMATION');
```



13.2 Library Functions

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%%% Example 13.2: Approximate values (x(i),y(i)), for i=1,2,...,N by a
%%%    polynomial f(x)=a(1)*x^M+a(2)*x^(M-1)+ ... a(M)*x+a(M+1) for M=1
% Definition of given values
    x=[0 0.2 0.5 0.7 0.8 1 1.2 1.6 1.9 2]';
    y=[5.2 4.3 4.4 4.9 5.5 6 6.2 8.4 8.9 10.9]';
% Evaluation of coefficients a and values of
% the approximation function
    a=polyfit(x,y,1), xx=[0:0.05:2]'; f=polyval(a,xx);
% Evaluation of the sum of square errors for given ranges
% of parameters a1, a2 and plot of results
    a1=2:0.1:4; a2=2.5:0.1:4.5;
    for i=1:21; for j=1:21
        S(i,j)=sum((a1(j)*x+a2(i)-y).^2); end; end
    subplot(2,2,2); plot(xx,f); hold on; stem(x,y); hold off;
        xlabel('x'); ylabel('y'); title('APPROXIMATION'); grid on
    subplot(1,2,1); mesh(a2,a1,S); grid on; xlabel('a2'); ylabel('a1'); title('ERROR SURFACE');
    subplot(2,2,4); contour(a2,a1,S); xlabel('a2'); ylabel('a1'); title('CONTOUR PLOT');
```

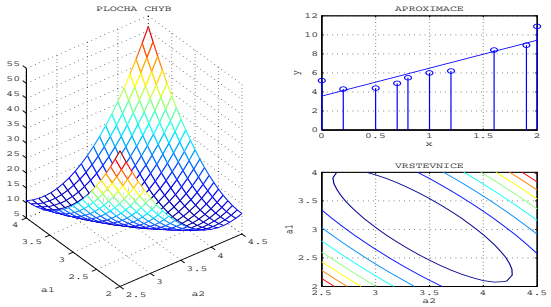

COMMANDS

SUM
INV

POLYFIT
POLYVAL

STEM
MESH
MESHC
CONTOUR

HOLD ON
HOLD OFF

EXAMPLES 13

13.1 Approximate the given sequence of values

$\mathbf{x}$=[0.3 0.4 0.6 0.9 1.5 2]';
$\mathbf{y}$=[11.7 10 8.3 7.2 6.3 6]';

by a function $f(x) = a(1)/x + a(2)$ and plot results

13.2 Approximate sequence of values

$\mathbf{x}$=[0 0.2 0.5 0.7 1.1 1.4 1.6 1.7 2]';
$\mathbf{y}$=[1 1.2 1.9 2.5 4.6 7.1 9.3 10.5 15]';

by polynomials of orders $M = 0, 1, ..., 5$ and plot dependence of the mean square error with respect to the polynomial order